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Abstract

Although a number of studies have attempted to test the hypothesis of a "Malthusian" pre-industrial world, few have focused on countries outside the UK, or on the "post-Malthusian" regime postulated by Unified Growth Theory. The present work is the first to test explicitly for the post-Malthusian regime in a setting outside the UK and Scandinavia, namely northern Italy from 1650. Employing a cointegrated VAR model, we find evidence that this part of the world does not fit cleanly into the Malthusian or post-Malthusian worlds, suggesting room for an extension of the simple Malthusian model.

JEL Codes: J1, N33, O4

Keywords: Cointegration, Italy, Malthusian, post-Malthusian

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1. Introduction

Recent years have witnessed something of a boom in studies attempting to test the hypothesis of a "Malthusian" pre-industrial world, a concept central to Unified Growth Theory (see Galor 2011). Such a world should be characterized by three relationships: 1) the preventive check, whereby fertility increases with income (possibly through marriages becoming more frequent at higher levels of income); 2) the positive check, whereby mortality varies inversely with income; and 3) diminishing returns to labor, i.e. the fall in income when the workforce/population increases, due to the existence of fixed factors of production, land in particular. Together, these relationships anticipate a stable "subsistence" level of income, whereby any increase in productivity and thereby income is wiped out by a resultant increase in population and the onset of diminishing returns. Unified Growth Theory posits that the third relationship should break down at some point, giving rise to a "post-Malthusian regime", which will eventually be superseded by a complete collapse of the Malthusian model as countries witness demographic transition and move into the world of modern economic growth. The present work is the first to test explicitly for the post-Malthusian regime in a setting outside the UK and Scandinavia (following the seminal work of Møller and Sharp 2014), namely northern Italy, and we find evidence that this part of the world does not fit cleanly into the Malthusian or post-Malthusian worlds, suggesting room for an extension of the simple Malthusian model to allow for specific economic and demographic behavior in this particular setting.

Examples of studies looking for evidence of Malthusian mechanisms in preindustrial societies are Bailey and Chambers (1993), Lee and Anderson (2002), Nicolini (2007), Crafts and Mills (2009) and Møller and Sharp (2014). The latter introduced a novel approach, using the cointegrated VAR model combined with data on real wages as well as birth, death and marriage rates, and a sound theoretical basis, to provide a way for testing for the post-Malthusian regime, and finding support for this from the sixteenth century until the late eighteenth century. Klemp and Møller (2016) apply a similar approach (although not using marriage rates) to Scandinavian data, finding similar results. Also very relevant is the work of Fernihough (2013), who also uses data from Northern Italy and, using a stationary VAR approach, finds evidence of both checks and diminishing returns to labor, although again without marriage rates.

We also look for evidence of Malthusian mechanisms in northern Italy as well, but in contrast to the work of Fernihough also allow for the preventive check to operate through the marriage rate. Moreover, our empirical approach follows Møller and Sharp (2014), and we use cointegration

and test explicitly for both the Malthusian and post-Malthusian regimes. This has the obvious advantage that our results are directly comparable with theirs as well as those of Klemp and Møller (2016). Although we find some of same dynamics as for England and Scandinavia, we also find some very different results. The rest of the paper is organized as follows. The following section presents the Malthusian model, and Section 3 presents our econometric framework. Section 4 presents the data and the historical context, Section 5 presents our empirical results, and Section 6 provides and interpretation of these. Finally, Section 7 concludes.

2. The Malthusian Model

In this section we present a simple model of Malthusian stagnation. First the simple model is presented without nuptiality, and then an augmented model including the marriage rates is presented following the work by Møller and Sharp (2014). The model considers a small, closed economy where the population size is endogenous. One homogenous good is produced using fixed land and labor as inputs. The income of each worker is represented by the real wage and labor is supplied inelastically at the market clearing wage in the aggregate labor market. Production is assumed to follow a constant-returns-to-scale Cobb-Douglas function. The following system of equations describe the Malthusian mechanisms:

$$w_t = c_0 - c_1 \ln N_t + \ln A_t \tag{1}$$

$$b_t = a_0 + a_1 w_t + \varepsilon_{bt} \tag{2}$$

$$d_t = a_2 - a_3 + \varepsilon_{dt} \tag{3}$$

$$\ln A_t = \ln A_{t-1} + \varepsilon_{At} \tag{4}$$

$$\ln N_t \equiv \ln N_{t-1} + b_{t-1} - d_{t-1} \tag{5}$$

w_t is the natural logarithm of the real wage, b_t is the crude birth rate, d_t is the crude death rate and N_t is the total population all at time t. A_t is an expression for the aggregate level of technology. The parameters of the model are all positive and the error terms are stochastic and normally distributed with zero mean and constant variances. The shocks to the demographic variables, ε_{bt} and ε_{dt}, represent unmodeled unsystematic influences on births and deaths.

Equation 1 is the wage expression and shows how real wages are affected negatively when the population/work force increases. When the level of technology changes the level of real wages changes as well. The parameter $c_1 > 0$ expresses the diminishing returns to labor. Equations 2 and 3 describe the preventive and positive checks respectively. The fact that the parameters $a_1, a_3 > 0$ indicates that the birth rate is affected positively by the real wage, while the death rate is affected negatively. In the literature the explanations for the preventive check are multiple, while the positive check can be explained by purely biological circumstances given the adverse effects of lower income on nutrition and mortality. There is some evidence that the preventive check, apart from being biologically determined is also a result of rational economic behavior of couples in terms of birth spacing and expected infant mortality; see for example Cinnirella et al. (2017) for pre-industrial England and Ejrnæs and Persson (2014) for a study on Italy. As shown in the next subsection, birth rates can also be explained by the number of marriages. Technology is expressed by equation 4 where it clearly follows a Random Walk process. This is an important assumption in the econometrical framework used later. Finally, equation 5 shows the dynamics of population growth. In the model the effects of external migration are ignored, and population growth is thus given by the difference between the birth and the death rates. The assumption of no external migration seems acceptable as it has been suggested that the effects of migration in northern Italy were insignificant until the second half of the nineteenth century (Galloway, 1994). The evolution of the labor force is assumed to be proportional to the total population.

Solving the system of equations with respect to $\{w_t, N_t, A_t, b_t, d_t)$ yields the short-run equilibrium. The steady state values can be found from the solution when population growth is zero, no shocks are present, and technology is held fixed. The resulting level of real wages is the subsistence level. The conditions for the existence of a steady state can be found in Møller and Sharp (2014).

Figure 1 illustrates the short-run equilibrium and the steady state levels for period t. The short-run equilibrium is illustrated with the dashed lines while the steady state is illustrated by the arrows.

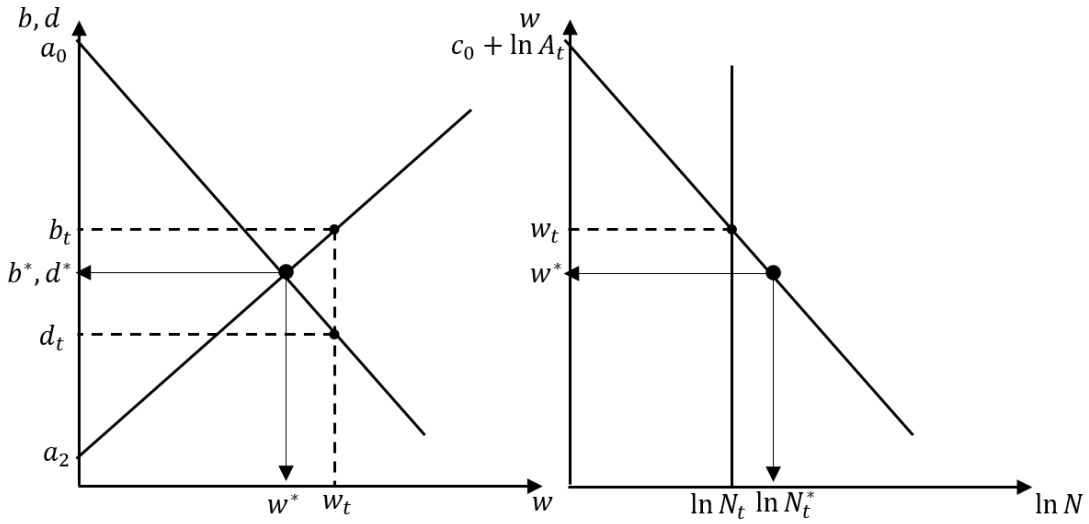

Figure 1 **- The standard Malthusian model.**
The left panel represents the preventive check (upward sloping) and the positive check (downward sloping). The right panel shows the demand (downward sloping) and supply (vertical) of labor for a given $\boldsymbol{A_t}$. The dashed lines represent the short-run equilibrium while the arrows indicate the steady state.

In the steady state the levels of w^*, b^* and $d^*$are no longer time dependent, while population still depends on time through the level of technology, A_t. The equilibrium starts in the right panel where the stocks of population and technology determine the short-run equilibrium level of real wages at time t. In the left panel the birth and death rates are determined adding their respective shocks, $\varepsilon_{b,t}$ and $\varepsilon_{d,t}$. In the absence of further shocks all variables will converge to their steady-state values in the subsequent periods, $\{w^*, \ln N_t^*, b^*, d^*\}$.

2.1 Augmenting with Marriage

Returning to the preventive check, an augmented model including nuptiality is suggested by Møller and Sharp (2014) where it is assumed that the preventive check works through marriages. In what follows we will briefly introduce the additional equations in the augmented model, but see Møller and Sharp (2014) for a complete discussion.

It is assumed that the number of births is positively correlated to the stock of fertile marriages denoted M_t^f which, in turn, is a weighted sum of the marriage rates in the last s years.

$$B_t = \vartheta M_t^f, \vartheta > 0 \tag{6}$$

$$M_t^f = \delta_1 M_{t-1} + \delta_2 M_{t-2} + \cdots + \delta_s M_{t-s} \tag{7}$$

The marriage rate, m_t, depends on future expected income and the relationship is derived assuming that expectations are adaptive of the form $w_{t+i}^e = w_t$. In conclusion the augmented model replaces equation 2 with the following two equations which describe the preventive check:

$$b_t \simeq e_0 + e_1 b_{t-1} - e_1 d_{t-1} + f_1 m_{t-1} + e_2 b_{t-2} - e_2 d_{t-2} + f_2 m_{t-2} + \cdots \tag{8}$$
$$+ \cdots + e_s b_{t-s} - e_s d_{t-s} + f_s m_{t-s} + \varepsilon_{bt}$$

$$m_t = a_4 + a_5 w_t + \varepsilon_{mt} \tag{9}$$

Equation 8 describes the birth-marriage part of the preventive check and equation 9 describes the marriage-income part. As in the simple model it is possible to derive both the short-run equilibrium and the steady state values.

The empirical analysis will be based on the augmented model, a choice supported both by the emphasis Malthus put on the marriage-based explanation of the preventive check, and because empirically including marriages improves the statistical specification of the birth rate equation. In the presence of Malthusian stagnation, the level of technology is assumed to be exogenous following a random walk. However, the empirical framework is constructed to test for the presence of post-Malthusian mechanisms. The post-Malthusian hypothesis posits that the diminishing returns to labor are no longer present in the standard model presented above while it is still possible to have the positive and preventive checks.

3. The Econometric Framework

Møller and Sharp (2014) explain how the Malthusian model presented above can be formalized as a cointegrated VAR model in the four known variables, $\{w_t, b_t, d_t, m_t\}$. Thus, all the estimations are general VAR(k)s in Error-Correction-Mechanism form:

$$\Delta x_t = \Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + \cdots + \Gamma_{k-1} \Delta x_{t-(k-1)} + \phi D_t + \varepsilon_t \tag{10}$$

Where $x_t = (w_t, b_t, d_t, m_t)'$ and D_t is a $d \times 1$ vector of dummies that initially will be left empty. The matrix Π contains the parameters of interest. If $\det(\Pi) \neq 0$ then x_t is stationary. Alternatively, if $\det(\Pi) = 0$ then x_t is non-stationary and Π has reduced rank, $r < 4$. This can be parametrized as a non-linear testable restriction:

$$\Pi = \alpha\beta' \tag{11}$$

Where α and β are $4 \times r$ matrices with $r < 4$.

The fact that $r < 4$ is a clear contradiction of the Malthusian model as derived in the previous section and indicates that the observable variables interact in a non-stationary VAR, motivating the cointegration approach applied here. The proposed Malthusian framework suggests two explanations for the presence of reduced rank. The first is that the sum of the positive and the preventive check is negligible, $a_1 + a_3 = 0$, resulting in almost no effect from the real wage to population growth. In this case the model has no equilibrium wage as the birth rate and death rate will never be equal (the two curves in the left panel of Figure 1 will be parallel). The second explanation, which can be related to the post-Malthusian hypothesis, is that real wages do not decline when the population increases, and thus there are no diminishing returns, and can be expressed as $c_1 = 0$.

To formulate the testable restrictions for the four observable rates we solve equations 1-5 and 8-9, with respect to the first differences of the four observables, Δw_t, Δb_t, Δd_t and Δm_t. Using the identity $x_t \equiv x_{t-1} + \Delta x_t$ the resulting equations can be written as a four-dimensional VAR(k) as in equation 12. The parameter matrix Π has the following long-run restriction for $s > 1$:

$$\Pi = \begin{pmatrix} 0 & -c_1 & c_1 & 0 \\ 0 & -(1-\bar{e}_1) & -\bar{e}_1 & f \\ -a_3 & a_3c_1 & -a_3c_1 - 1 & 0 \\ a_5 & -a_5c_1 & a_5c_1 & -1 \end{pmatrix} \tag{12}$$

Where $\bar{e}_1 \equiv \sum_{i=1}^{s} e_i$ and $f = \sum_{j=1}^{s} f_j$. Letting $a_1^* = a_5 f$ the determinant of equation 12 is:

$$\det(\Pi) = c_1(a_1^* + a_3) \tag{13}$$

From equation 13 it is clear that when $c_1 = 0$ and/or $a_1 + a_3 = 0$, the determinant is zero and thus we have reduced rank, in which case the restriction can be written as $\Pi = \alpha\beta'$.

To be able to apply the theory to the data it is however necessary to use a more flexible version of the model, which allows for dynamic adjustments, but maintains the same long-run properties. The determinant of the full rank long-run restriction becomes:

$$\det(\Pi) = c_1(\tilde{a}_1 + \tilde{a}_3)(1-\lambda)(1-\delta)(1-\lambda-\phi) \tag{14}$$

provided that $(1-\lambda) \neq 0$, $(1-\delta) \neq 0$, $(1-\lambda-\phi) \neq 0$ and

$$\tilde{a}_1 \equiv \tilde{a}_5\tilde{f}\ ,\ \tilde{f} \equiv \frac{\sum_{i=0}^{k} f_i}{(1-\lambda-\phi)}\ ,\ \tilde{a}_3 \equiv \frac{-\psi}{1-\lambda}\ ,\ \tilde{a}_5 \equiv \frac{\eta}{(1-\delta)} \tag{15}$$

The long-run parameters in this version are the same as in the standard model and can thus be read directly from the regression analysis. When equation 14 has reduced rank, we obtain two reduced rank restrictions to be tested empirically. The first is when $\tilde{a}_1 + \tilde{a}_3 = 0$, and the two identified cointegrating vectors are:

$$\alpha = \begin{pmatrix} 0 & 0 & -c_1 \\ -f_0(1-\delta) & -(1-\gamma-\phi) & -\phi - \eta_0 c_1 f_0 \\ -\tilde{f}(1-\lambda) & -(1-\lambda) & 1-\lambda-\psi_0 c_1 \\ -(1-\delta) & 0 & -\eta_0 c_1 \end{pmatrix} \tag{16}$$

$$\beta' = \begin{pmatrix} -\tilde{a}_5 & 0 & 0 & 1 \\ 0 & 1 & 0 & -\tilde{f} \\ 0 & 1 & -1 & 0 \end{pmatrix}$$

The second case is when $c_1 = 0$, with cointegrating vectors:

$$\alpha = \begin{pmatrix} 0 & 0 & 0 \\ \frac{(\phi\tilde{a}_3 - \eta f_0)}{\tilde{a}_5} & -(1-\gamma) & \phi \\ 0 & 0 & -(1-\lambda) \\ -(1-\delta) & 0 & 0 \end{pmatrix} \tag{17}$$

$$\beta' = \begin{pmatrix} -\tilde{a}_5 & 0 & 0 & 1 \\ 0 & 1 & 0 & \frac{\phi\tilde{a}_3}{(1-\gamma)} - \frac{(1-\gamma-\phi)}{(1-\gamma)}\tilde{f} \\ \tilde{a}_3 & 0 & 1 & 0 \end{pmatrix}$$

In both cases the β is identified, see Møller and Sharp (2014). In section 5 the hypotheses will be tested on the northern Italian data.

4 Data and Context

Our analysis relies on four data series, comparable to those for England used by Møller and Sharp (2014). The sources are the same as those used by Fernihough (2013), but we also introduce the crude marriage rate along with the birth and death rates. The demographic series come from Galloway (1994) and give annual observations of crude birth, marriage and death rates (per thousand head of population). The real wage data (here given in logarithms) come from

Malanima (2007) and represent average daily wages covering both rural and urban areas.[3] For the urban wages, masons have been used and the rural wages represent laborers. As a robustness check, we also use the inverse of wheat prices and perform the analysis using two distinct measures of real wages: agricultural wages and urban wages. These checks, however, make little difference to our conclusions.[4]

The demographic series cover an area labelled "Northern Italy" including the five regions: Lombardy, Piedmont, Tuscany, Veneto and Emilia-Romagna, see Figure 2. Data on the wage rate originate from the regions of Tuscany for the urban wages and Lombardy/Tuscany for the rural wages. A concern regarding the real wages is that they might not be representative for the whole area labeled "Northern Italy". However, the urban wages of Tuscany are highly correlated with comparable wages for the other regions and the rural wages are assumed to be similar to the rest of northern Italy, as evidence suggests that changes in conditions of the laborers were similar in the various regions (Federico and Malanima, 2004).[5] Furthermore, we use the real wages covering both urban and rural areas in our main analysis because of the expansion of the urban economy that caused relative urban wages to rise during the sixteenth century. Thus, using only the urban wages would bias the results.

Before 1861 the Italian regions were under different governments and thus studying separately the different regions (because of lack of demographic series for the south and a wage rate covering the entire peninsula) does not create any issues. In fact, it can be argued that there might be important differences between north and south (see Ejrnæs and Persson 2014), justifying a focus on northern Italy alone.

[3] Available online: http://www.paolomalanima.it/default_file/Italian%20Economy/Wages_Italy_1290_1990.pdf
[4] The robustness checks can be found in Appendix 2. We also considered using GDP data as a robustness check. However, these estimates (Malanima 2011) are based on real wages and thus would not be independent check.
[5] The data in Malanima does not include these other regions because of the shorter period covered.

***Figure 2* – Northern Italy**

As in the rest of Europe, urban wages in Northern Italy increased after the Black Death, and the level remained relatively high for about a century. Around 1500, Northern Italy was the most advanced European region together with the Netherlands. Wages started decreasing during the second half of the fifteenth century and reached a minimum after 1530. Thereafter, the position of Northern Italy in the European hierarchy weakened. In the early seventeenth century, shortly before our complete dataset starts, the Netherlands were already much wealthier than Italy, while Spain also enjoyed a higher level of per capita GDP. Therefore, the economic dominance that Italy had enjoyed during the renaissance was gone at the time where our analysis starts. Figure 3 graphs GDP per capita for Italy, England, the Netherlands and Sweden and makes it clear that the Netherlands had a higher GDP per capita already before 1650 and the English GDP per capita become higher than the Italian after 1700.[6]

[6] However, Malanima (2013) argues that the Italian GDP was overtaken by the English only after the second half of the eighteenth century. In figure 3 we use data from Maddison for all four countries because it makes it easier to compare them, even though other measures are available.

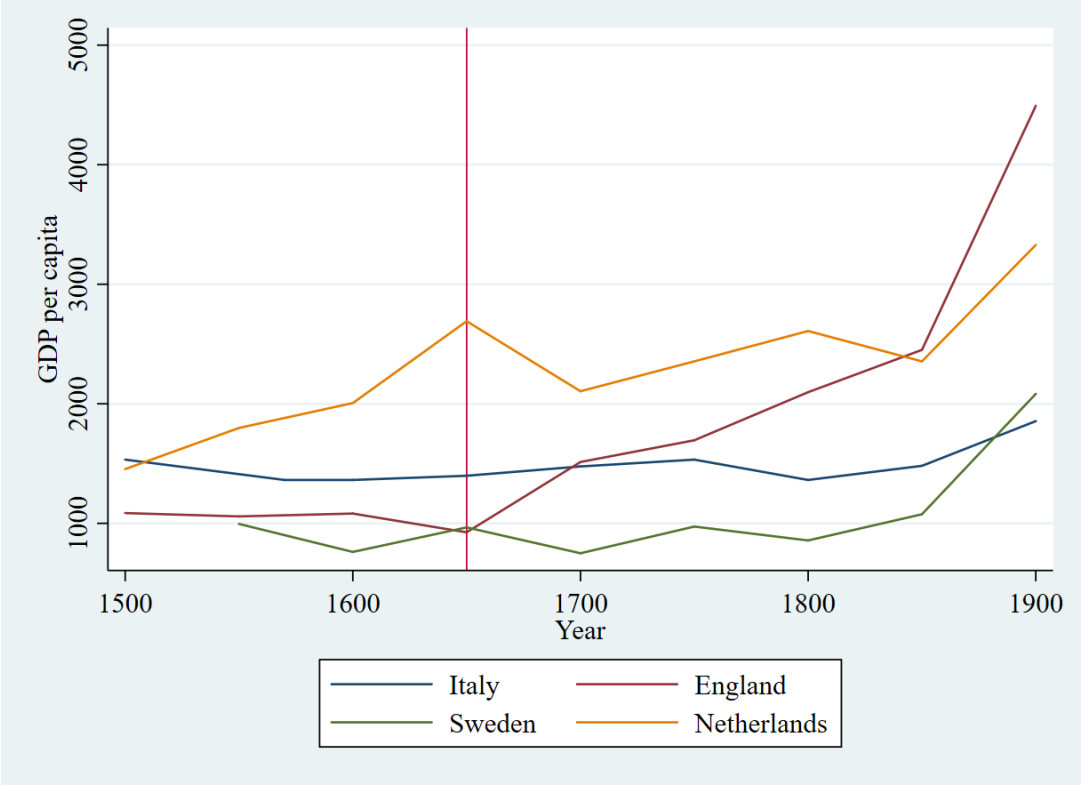

***Figure 3* – GDP per capita for Italy, England, Sweden and the Netherlands in 1990 international GK$.**
Sources: Bolt and van Zanden (2014). The vertical line indicates when our dataset for the analysis starts.

Before proceeding it is also important to consider that in 1629-30 most of Northern Italy suffered from a severe plague causing the population to decrease and real wages to increase for a period that might affect some of the first observations in our dataset which start in 1650. We nevertheless initially concentrate on the years from 1650 (when the demographic data starts) until 1799. The latter date is arbitrarily chosen for convenience to be sure to exclude changes associated with industrialization. However, as a control, the sample is extended later in the analysis using recursive regressions, with the effect that the results become less significant.

Figure 4 illustrates our data in both levels and differences. The first point to notice is that real wages have a negative trend over time, which distinguishes northern Italy from other regions where the same approach has been used, i.e. England and Scandinavia. The trend becomes even more clear when using a longer time series of real wages.[7] Malanima (2003, 2006) explains this in part by stagnating nominal wages combined with increases in the price index and bad weather. A concern about using the real wage rate is that it does not consider that, when the wage rate falls, workers will

[7] Malanima's complete series starts in 1290.

increase the hours worked, or women and children will join the working force, to compensate the decrease and keep income at the same level.[8] However, real GDP estimates, which since they are on an annual basis are insensitive to the number of days worked, still exhibit a negative trend, although less pronounced than for the real wage rate.

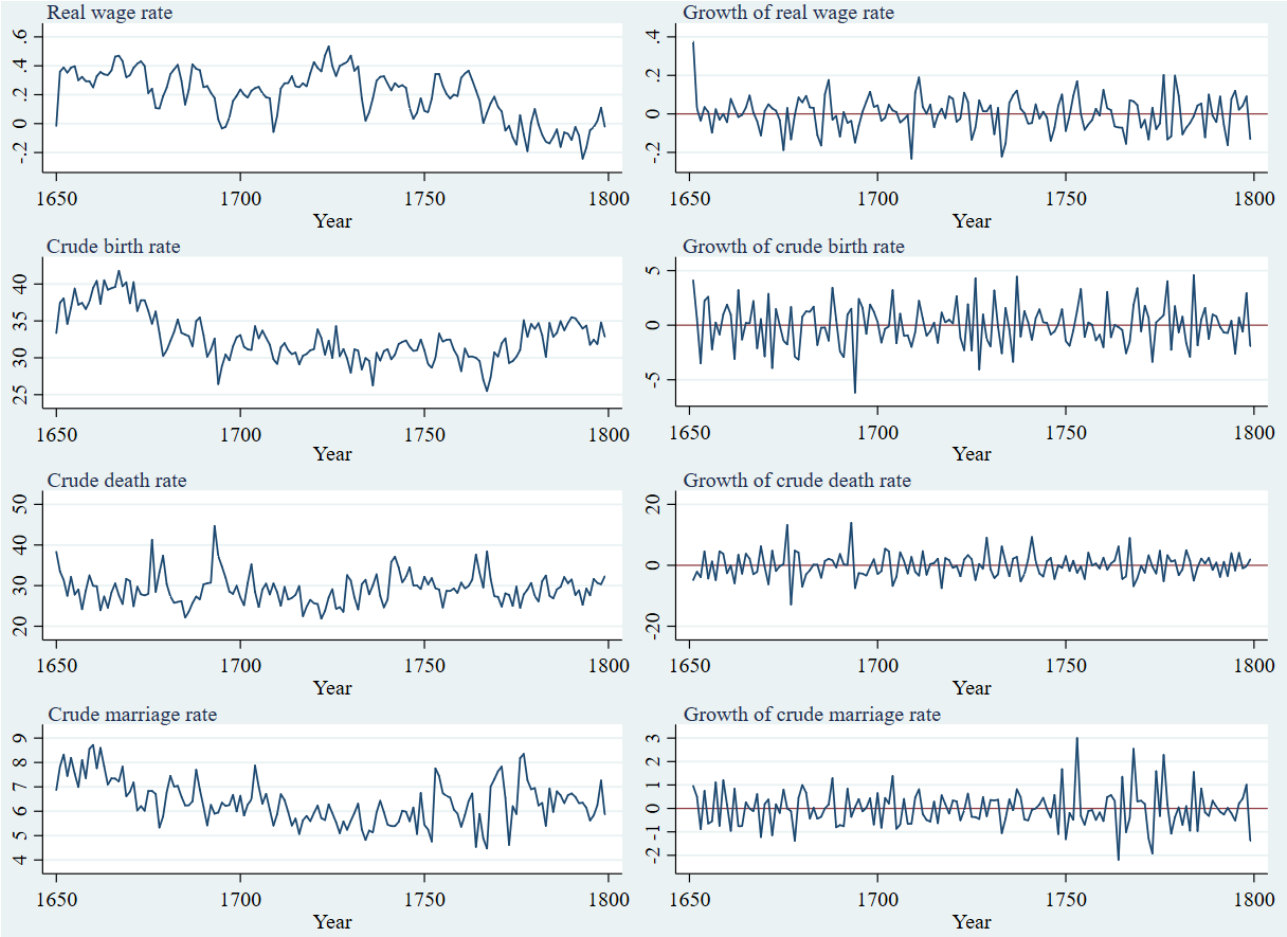

***Figure 4* – Real wages and vital rates in levels and differences (1650-1799).**

Sources: The time series for real wages has been collected and estimated by Paolo Malanima and the vital rates all come from Galloway (1994a).

Furthermore, both marriage rates and birth rates also seem to decrease over time, while the death rate seems oscillating around some stable value for the entire period represented. Finally, it can be noticed from the graphs that lw, cbr, and cmr all seem non-stationary, while their first-differences appear much more stationary. cdr, on the other hand, appears more stationary in levels.

[8] According to Malanima (2006), there was indeed an increase in the working hours in northern Italy along with a rise in non-labor incomes.

Since we will be looking for cointegrating relationships between the variables, the order of integration needs to be considered for the correct interpretation of the results. While most of the variables might be approximated well by an I(1) processes, *cdr* seems to be I(0), even though the test-statistics for all the variable are very sensitive to the time interval chosen and the number of lags included.[9] If *cdr* is indeed I(0) we should not expect to find cointegrating relations including this in the empirical analysis. But using another econometric approach could, in any case, result in spurious regression results in case the included variables are of different orders of integration. We therefore proceed with the analysis following the cointegration approach.

5 Analysis

5.1 Preliminary Analysis

To have a well-specified model there are several things to consider before estimating the final CVAR.[10] Following Møller and Sharp (2014) we start by estimating an unrestricted VAR:

$$\Delta x_t = \Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + \cdots + \Pi_{k-1} \Delta x_{t-(k-1)} + \phi D_t + e_t \tag{18}$$

where $x_t' = (w_t, b_t, d_t m_t)$, e_t is gaussian iid and ϕD_t includes an unrestricted constant, a restricted linear trend and possibly some dummy variables to control for outliers and assure normality in the residuals. The analysis is also dependent on the correct choice of the lag length, to avoid issues regarding autocorrelation. In a pre-estimation test we find that including three lags is the best choice.[11] In contrast to the two lags often found sufficient for analyzing macroeconomic data (see the discussion in Juselius 2006), it seems that demographic data requires an extra lag, since three lags were also found necessary when modelling the English data by Møller and Sharp (2014) and also for Italy by Fernihough (2013).

An analysis of the residuals reveals, as expected, no problems with autocorrelation in the unrestricted VAR model. However, the assumption that the residuals are normally distributed is convincingly rejected. Univariate analyses of the variables reveal that non-normality is mainly due to the non-normality of *lw* and *cdr*. For the former, this is caused primarily by one large negative residual (-3.52) in the year 1709. This is probably due to the outcome of the War of the

[9] Augmented Dickey-Fuller Test-Statistics for the presence of a unit root have been computed with up to 5 lags, see Appendix 1.
[10] The regression analysis has been made in PcGive, OxMetrics 4.02 and STATA 15.1.
[11] The results from the pre-estimation analysis are presented in Appendix 1.

Spanish Succession, when northern Italy came under the Austrian crown. Non-normality of *cdr* is primarily the result of two larger positive residuals (2.90) in 1676 and (4.28) in 1693.[12] The former is attributable to famine while the latter is attributable to an outbreak of smallpox. We model all three as exogenous shocks using dummies of the form $(\dots 0001000 \dots)$. Including these dummies considerably improves the model specification going from being normally distributed with a probability of less than 1% to being normally distributed with a probability of 7.39%. Nevertheless, problems with non-normality and autoregressive conditional heteroskedasticity have been proved to be less severe for the cointegration analysis than problems with autocorrelation (Juselius 2006, Rahbek *et al.*, 2002), the latter of which is not an issue when using three lags.

When including dummies in the analysis, it is important to decide whether to include them unrestricted or restricted in the cointegration model. A distinction can be made between innovational outliers (IO) which are due to extraordinary exogenous shocks such as warfare and epidemics and additive outliers (AO) which are due to measurement errors. The former should enter the model unrestricted while the latter should enter restricted in the cointegrating equations. As all three identified outliers have been caused by well-documented episodes of exogenous shocks we choose to include them as unrestricted in the analysis.[13]

Finally, before testing the two restrictions in equations 16 and 17, we test the rank of the long-run restriction, Π, i.e. the number of possible cointegrating relations. In the specification including the three dummy variables, $D1676$, $D1693$ and $D1709$, the rank is clearly suggested to be $r = 3$, indicating that the rates are persistent and thus not consistent with the Malthusian model described in section 2. As a robustness check of the non-stationarity, and to assess when the non-stationarity becomes empirically relevant, we proceed with a backward recursive analysis of the trace test using a fixed endpoint in 1799. This exercise is done both with and without the dummy variables included, to determine how these influence the choice.

Figure 4, panel A shows the p-values from the recursively estimated trace test of $H(3)$ against $H(4)$ i.e. the hypothesis that $r \leq 3$ against $r \leq 4$: when the trace test is statistically significant it is evidence against $H(4)$. From Figure 4, panel A it appears clearly that $r \leq 3$ for almost all recursions in both

[12] Usually, according to Hendry and Juselius (2001) an outlier is defined by a standardized residual exceeding 3.3. However, the inclusion of a dummy controlling for 1676 improves the specification considerably and we thus choose to include it even though the residual is less than 3.3.

[13] Including the dummies restricted in the analysis does not change the results much. The rank determined by the trace test is the same as in the unrestricted case, and the parameters of the cointegrating matrices are almost identical.

cases, as the p-values are almost always above 10%. To assess whether it is reasonable that $r = 3$ we do the same exercise for the trace test of $H(2)$ against $H(3)$, see figure 4 panel B. Since the p-values in this case are rather small, especially for the earlier sample starting points we are confident that $r = 3$.[14]

From figure 4 it is also evident that the trace test is influenced by the inclusion of dummies, but not to a large extent, since the graphs are very similar. Finally, it might be noticed that the non-stationarity is empirically relevant already from the initial years, as the p-values are high from the beginning. Given that we find evidence for $r = 3$, we thus proceed by testing the two proposed restrictions in 16 and 17.

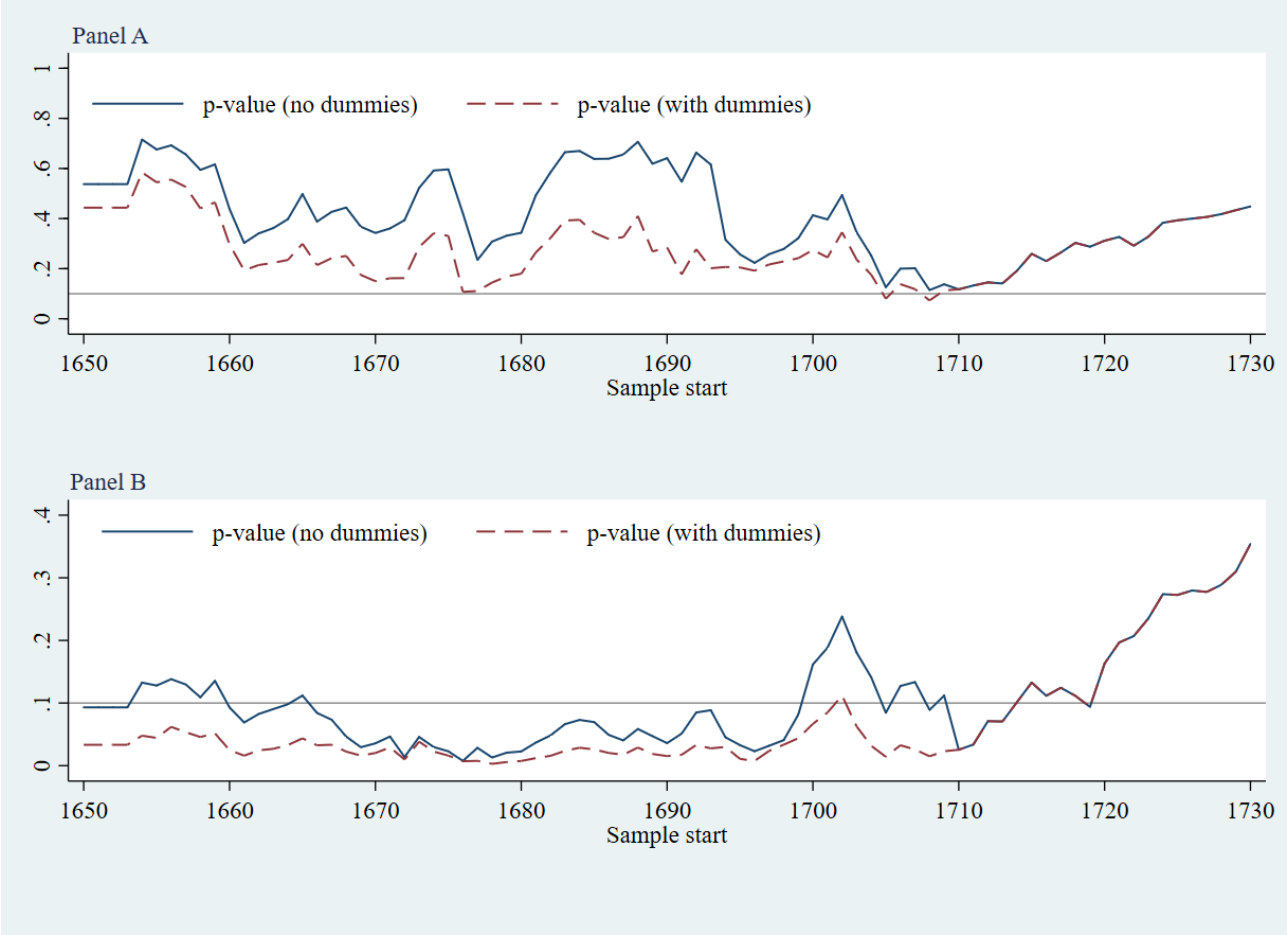

***Figure 5* – Recursively calculated trace test.**
Panel A presents the $\boldsymbol{p}$-values for the trace test for $\boldsymbol{H(3)}$ against $\boldsymbol{H(4)}$ and panel B presents the $\boldsymbol{p}$-values for $\boldsymbol{H(2)}$ against $\boldsymbol{H(3)}$. The solid line represents the model without dummies included, the dashed line represents the model where dummies are included, and the horizontal line indicates the 10% significance level. The $\boldsymbol{p}$-values have been computed using the Γ-approximation from Doornik and Hendry (1998). The sample end- point is fixed in 1799.

[14] This is most true for the model including the dummy variables. Apart from the trace test we also used the information criteria and the maximum eigenvalue statistic for the determination of the rank and they all support this finding.

5.2 Cointegration Analysis

The results from testing the restriction in equation 16 can be seen in Table 1 and in equation 19, while the results of the post-Malthusian hypothesis, equation 17, are found in Table 2 (p-values in parentheses) and equation 20. In the final specifications, the trend has been restricted to zero when its parameter was not significantly different from zero while the dummies enter unrestricted in all specifications.

In Table 1, we have replaced the positive check with the assumption of homeostasis (i.e. that in equilibrium $a_1 + a_3 = 0$). The first cointegrating relationship, $\beta_1{}'$, represents the preventive check, the second, $\beta_2{}'$, represents the birth relation and the third, $\beta_3{}'$, represents the homeostasis assumption. Even though the parameters in the cointegrating vector (preventive check and birth relation) and most of the adjustment parameters are significantly different from zero, the restriction cannot be accepted at any conventional level given a p-value of 0.000.

***Table 1* – CVAR regression results for the specification considering homeostasis for the sample 1650-1799**

α			β'					
				w_t	b_t	d_t	m_t	$Trend$
Δw_t	0.000	0.000	−0.001	β_1'	1.441	0.000	0.000	1.000	0.001
			(0.365)		(0.067)				(0.003)
Δb_t	0.296	−0.204	−0.040	β_2'	0.000	1.000	0.000	−2.534	0.000
	(0.174)	(0.001)	(0.184)					(0.000)	
Δd_t	1.567	0.423	−0.350	β_3'	0.000	1.000	−1.000	0.000	0.000
	(0.000)	(0.001)	(0.000)						
Δm_t	−0.349	0.000	−0.012						
	(0.000)		(0.349)						
Number of obs.		147							
LR test of identifying restrictions $chi^2(5)$		24.88 (0.000)							

Note: The brackets contain the p-values.

$$\begin{pmatrix}\Delta w_t\\ \Delta b_t\\ \Delta d_t\\ \Delta m_t\end{pmatrix}=\begin{pmatrix}0.000 & 0.000 & -0.001\\ 0.296 & \mathbf{-0.204} & -0.040\\ \mathbf{1.567} & \mathbf{0.423} & \mathbf{-0.350}\\ \mathbf{-0.349} & 0.000 & -0.012\end{pmatrix}\begin{pmatrix}\{\mathbf{1.441}w_t+m_t+\mathbf{0.001}t\}\\ \{b_t-\mathbf{2.534}m_t\}\\ \{b_t-d_t\}\end{pmatrix}+\cdots \quad (19)$$

In Table 2 weak exogeneity is imposed on real wages while both checks and the birth relation are present. The weak exogeneity comes from the first row in the adjustment matrix, $\alpha_{1,i}$, meaning that wages do not adjust after a shock in one of the other variables. The cointegrating relations are the same as before apart from the third, $\beta_3{}'$, which has been substituted with the positive check relation. Again, the restriction cannot be accepted given a very small p-value of 0.001. The parameters of the preventive check and the birth relation are both significantly different from zero, whereas the positive check is not (this was to be expected as the death rate appeared to be I(0)). All parameters in the adjustment matrix have the expected sign and are significant.

***Table 2* – CVAR regression results for the specification considering the post-Malthusian hypothesis for the sample 1650-1799**

α				β'					
					w_t	b_t	d_t	m_t	$Trend$
Δw_t	0.000	0.000	0.000	β_1'	2.558 (0.022)	0.000	0.000	1.000	0.011 (0.012)
Δb_t	0.000	−0.258 (0.000)	−0.076 (0.068)	β_2'	0.000	1.000	0.000	−4.083 (0.000)	0.000
Δd_t	0.000	0.000	−0.540 (0.000)	β_3'	2.499 (0.293)	0.000	1.000	0.000	0.000
Δm_t	−0.342 (0.000)	0.000	0.000						
Number of obs.		147							
LR test of identifying restrictions $chi^2(5)$		28.97 (0.001)							

Note: The brackets contain the p-values.

$$\begin{pmatrix}\Delta w_t\\ \Delta b_t\\ \Delta d_t\\ \Delta m_t\end{pmatrix}=\begin{pmatrix}0.000 & 0.000 & 0.000\\ 0.000 & \mathbf{-0.258} & \mathbf{-0.076}\\ 0.000 & 0.000 & \mathbf{-0.540}\\ \mathbf{-0.342} & 0.000 & 0.000\end{pmatrix}\begin{pmatrix}\{\mathbf{2.558}w_t+m_t+\mathbf{0.011}t\}\\ \{b_t-\mathbf{4.083}m_t\}\\ \{2.499w_t+d_t\}\end{pmatrix}+\cdots \quad (20)$$

From the above regression results, it can be concluded that none of the presented theoretical explanations of the reduced rank, can be accepted, indicating that the Italian data exhibit dynamics not included in the Malthusian model from section 2, including the restrictions compatible with a post-Malthusian regime. Extending the analysis to include the complete data set (1650-1881) does not change this conclusion. An inspection of Tables 1-2 also reveals another interesting aspect. In both, the parameter on the marriage rate ($\tilde{a}_5$) does not have the expected sign. At first glance, this seems to indicate that marriages are influenced negatively by real wages, i.e. that higher wage rates lead to fewer marriages and consequently to less births, which seems highly contradictory to both what we might expect from the period under consideration and the theory presented. We will return to this issue in section 6.

To get a greater understanding of why neither of the restrictions are accepted, we proceed by relaxing the restrictions on the adjustment parameters. In the first case (the homeostasis assumption), the restriction is still rejected while in the second case (weakly exogenous wages) we find a specification that can be accepted with a high p-value. We find that relaxing the restrictions on $\alpha_{1,i}$, can be accepted and as can be seen in Table 3, two of the three adjustment parameters are significantly different from zero (adjustment parameters of wages in the preventive and positive check relations). This indicates that these were the reason for rejection of the results in Table 2. Furthermore, the preventive check and the birth relation parameters are still significant with the same sign, while the positive check is insignificant.

***Table 3* – CVAR regression results for the alternative specification without weakly exogenous wages for the period 1650-1799**

α				β'					
					w_t	b_t	d_t	m_t	$Trend$
Δw_t	-0.045	-0.004	-0.006	β_1'	5.244	0.000	0.000	1.000	0.021
	(0.000)	(0.206)	(0.014)		(0.000)				(0.000)
Δb_t	0.000	-0.250	-0.068	β_2'	0.000	1.000	0.000	-4.664	0.000
		(0.000)	(0.101)					(0.000)	
Δd_t	0.000	0.000	-0.543	β_3'	2.600	0.000	1.000	0.000	0.000
			(0.000)		(0.268)				
Δm_t	-0.282	0.000	0.000						
	(0.000)								
Number of obs.		147							
LR test of identifying		6.284							
restrictions $chi^2(5)$		(0.507)							

Note: The brackets contain the p-values

$$\begin{pmatrix} \Delta w_t \\ \Delta b_t \\ \Delta d_t \\ \Delta m_t \end{pmatrix} = \begin{pmatrix} \mathbf{-0.045} & -0.004 & \mathbf{-0.006} \\ 0.000 & \mathbf{-0.250} & -0.068 \\ 0.000 & 0.000 & \mathbf{-0.543} \\ \mathbf{-0.282} & 0.000 & 0.000 \end{pmatrix} \begin{pmatrix} \{\mathbf{5.244}w_t + m_t + \mathbf{0.021}t\} \\ \{b_t - \mathbf{4.664}m_t\} \\ \{2.600w_t + d_t\} \end{pmatrix} + \cdots \qquad (21)$$

The conclusion that two parameters in the first row of the adjustment matrix necessarily must be different from zero indicates clearly that wages are not weakly exogenous in the system pointing towards an economy which is more Malthusian than post-Malthusian. The restrictions are accepted with a p-value of 0.507.

As mentioned in section 4 we have also conducted the analysis using two different measures of real wages to investigate whether there are some important differences between rural and urban areas, as they in periods exhibit markedly different movements. However, both show similar results and the

sign on the wage rate in the marriage equation is always the same. Our findings are also insensitive to using the inverse of wheat prices, indicating that the results presented in Tables 1-3 are robust to the choice of wages used.

5.3 Recursive Estimates

As a final step in the empirical analysis, we present the results of a forward-recursive regression to show how the results from the previous section change over time and when including the complete series available. In the forward-recursive analysis, one more observation is added in each recursion and it allows us to investigate changes in the parameters of interest and the test statistic of rejection. However, the recursive estimation keeps all the observations from the earlier years and thus tends to underestimate potential changes later in the sample. We have therefore also run rolling windows analysis based on 70-years rolling subsamples to see whether there are relevant changes at the end of the sample. The analysis has been done for all three specifications from section 5.2 and the recursive graphs can be seen in Figures 6-8. All parameters converge and in the preventive check is always with the unexpected sign indicating that this result is robust to the choice of any subsample. In Figure 7 the preventive check oscillates more at the beginning, when including only the first years, but when more observations are included the parameter again converges and stays positive. The parameter of the birth relation always has the same sign and oscillates only a little, while the positive check parameter oscillates more exhibiting a negative value at one point but converging as the sample size is increased. The last panel in each figure shows the p-values of the restrictions. In Figures 6-8 the restrictions can almost always be rejected indicating again, that the results from section 5.2 are robust to the choice of the observations included. In Figure 8 the p-value varies more but is often higher than 5% and especially at the beginning of the sample we obtain very high values. When including more observations from the end of the sample (starting from around 1815) the p-value converges to a low value and the specification is always rejected.

The results from the rolling windows analysis can be seen in the Appendix 2. When using 70 year subsamples the conclusions are much the same. In all three cases most of the subsamples can reject all three specifications apart from a few starting from 1750 which can accept the homeostasis hypothesis and some subsamples accepting the final specification from Table 3.

From the above analysis it can be concluded that there are no clear cut-offs where the dynamics change for any of the three specifications apart from the end of the sample when the restrictions can never be accepted.

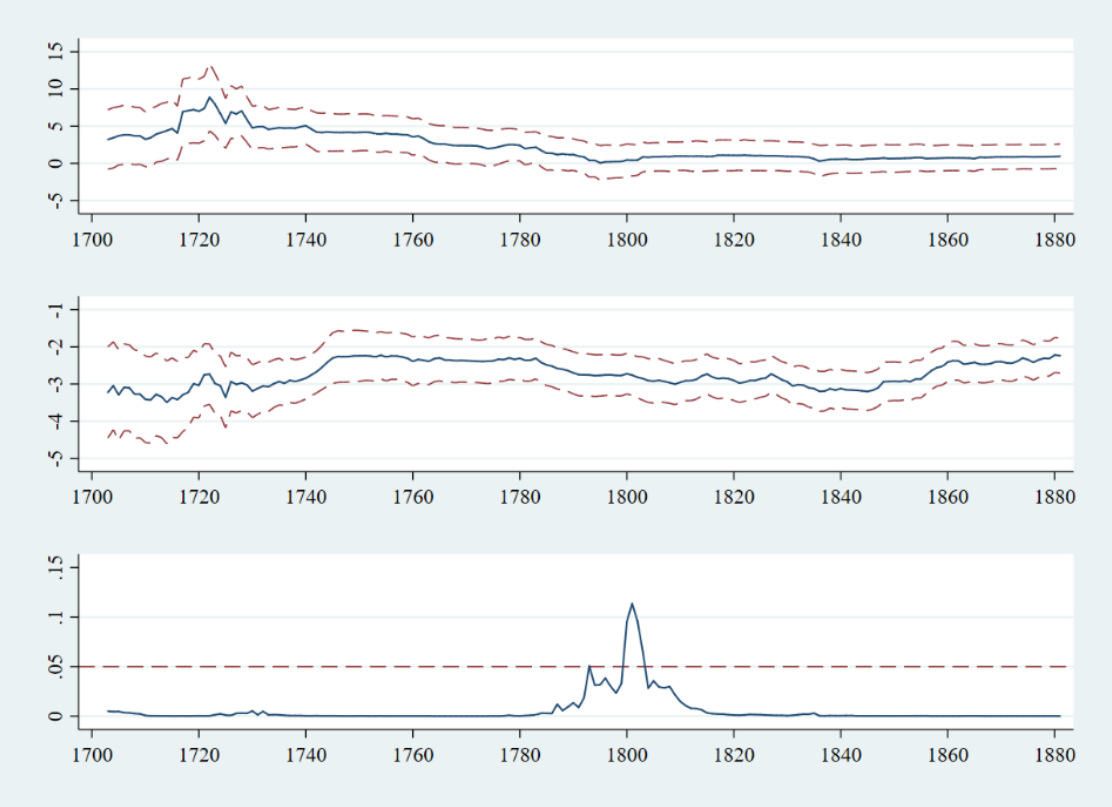

***Figure 6* – Recursive estimates for the specification considering homeostasis for the sample 1650-1881.**

Notes: (top panel) the point estimate of the preventive check coefficient, stated as $\boldsymbol{\alpha_5}$, together with the 2x Standard Error limits; (middle panel) point estimate of the birth relation coefficient together with the 2x Standard Error limits; (bottom panel) the $\boldsymbol{p}$-values of the test statistic corresponding to the homeostasis hypothesis, where values below the dashed line indicate a rejection of the hypothesis at the 5 percent significance level. Sample start is fixed at 1650.

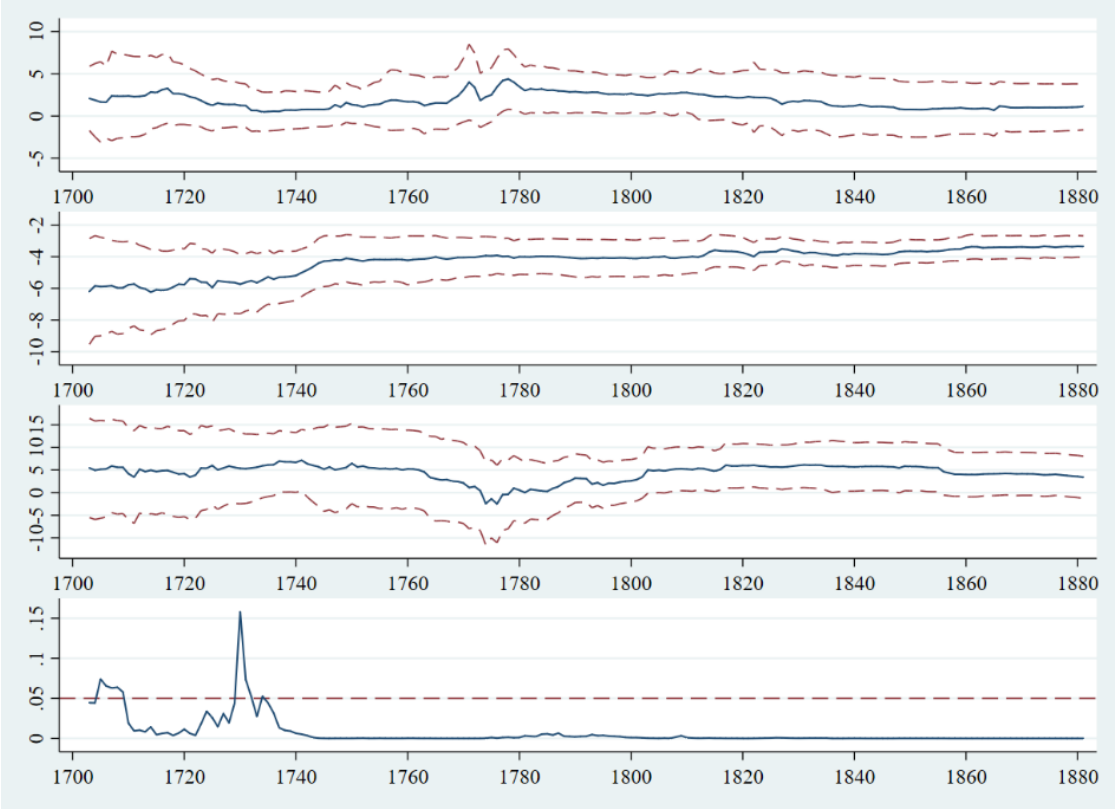

***Figure 7* – Recursive estimates for the specification considering the post-Malthusian hypothesis for the sample 1650-1881.**

Notes: (top panel) the point estimate of the preventive check coefficient, stated as $\boldsymbol{a_5}$, together with the 2x Standard Error limits; (top middle panel) point estimate of the birth relation coefficient together with the 2x Standard Error limits; (middle bottom panel) the point estimate of the positive check coefficient, stated as $\boldsymbol{a_1}$, together with the 2x Standard Error limits; (bottom panel) the $\boldsymbol{p}$-values of the test statistic corresponding to the homeostasis hypothesis, where values below the dashed line indicate a rejection of the hypothesis at the 5 percent significance level. Sample start is fixed at 1650.

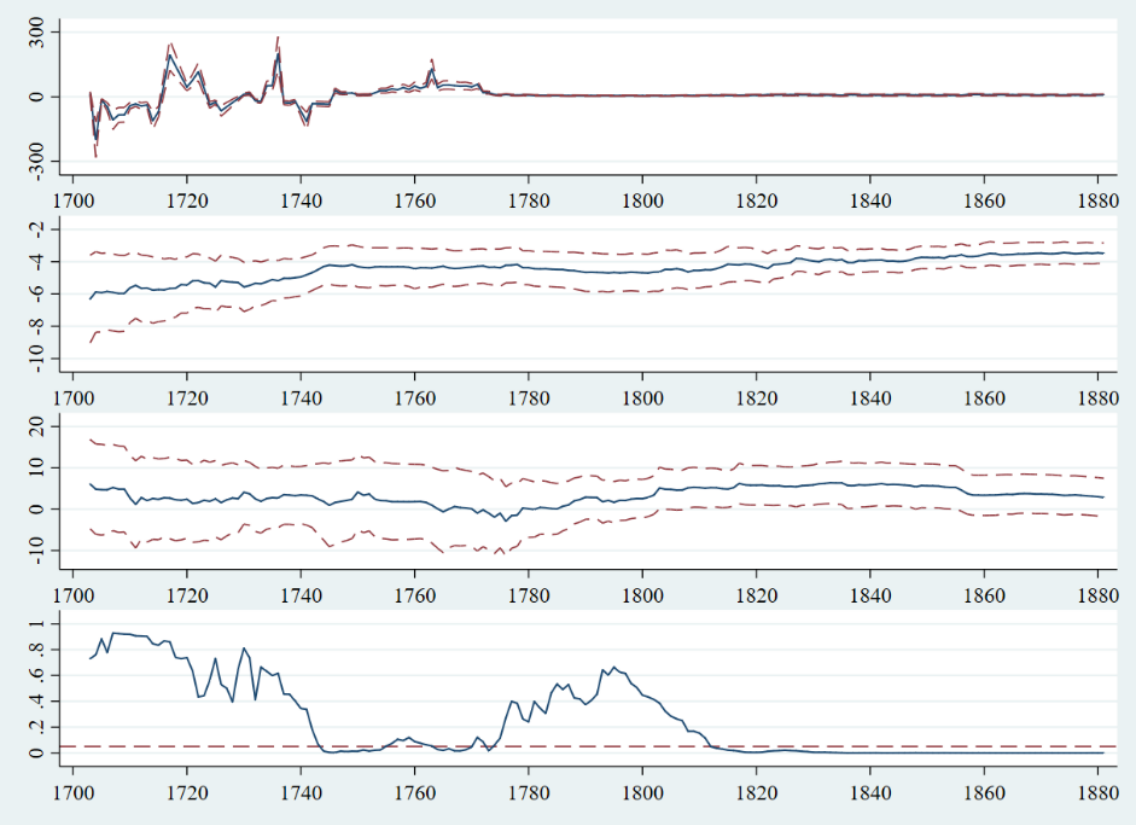

***Figure 8* – Recursive estimates for the alternative specification without weakly exogenous wages for the period 1650-1799.**

Notes: (top panel) the point estimate of the preventive check coefficient, stated as $\boldsymbol{a_5}$, together with the 2x Standard Error limits; (top middle panel) point estimate of the birth relation coefficient together with the 2x Standard Error limits; (middle bottom panel) the point estimate of the positive check coefficient, stated as $\boldsymbol{a_1}$, together with the 2x Standard Error limits; (bottom panel) the $\boldsymbol{p}$-values of the test statistic corresponding to the homeostasis hypothesis, where values below the dashed line indicate a rejection of the hypothesis at the 5 percent significance level. Sample start is fixed at 1650.

6 Discussion and the preventive check

In all three specifications the sign of the preventive check is found to indicate that increases in the real wage cause a decrease in the marriage rate and consequently a decrease in the number of births. This result is in contradiction with the Malthusian preventive check and might be interpreted as evidence in favor of the quantity-quality trade off emerging during the process of industrialization. However, the sign is the same for almost all subsamples in both the recursive and rolling windows analysis, indicating that the negative relationship starts much earlier than the process of industrialization. It thus seems implausible to claim the existence of a quantity-quality trade off based on the present analysis. It is also contradictory to the results found by Fernihough (2013) where there

is evidence supporting the preventive check with the expected sign even though using a different approach. Furthermore, other studies following the cointegration approach find evidence for a preventive check with the expected sign (Møller and Sharp 2014 and Klemp and Møller 2016).

On the other hand, Chiarini (2007) finds no evidence for preventive checks. In fact, like us, he finds that neither fertility nor mortality respond to real wages, and he even finds evidence for diminishing returns for a long time series on population and wages (thirteenth to nineteenth centuries). We find that diminishing returns are driven by changes in fertility rather than mortality, and that is indeed the natural conclusion to be drawn from Galloway (1994), who noted that changes in the population growth rate were dominated by fluctuations in fertility. Most significantly, our finding of declining income due to diminishing returns supports the claims of several scholars. Malanima (2003, 2006) suggests that the decline in real wages was partly due to diminishing returns – after all, population doubled between 1700 and 1861 - but he also gives other reasons, most significantly a fall in temperatures in the second half of the eighteenth and the first two decades of the nineteenth century. Since more than 40 percent of Italy's surface is hilly, Italian agriculture was hit particularly hard by the temperature drop, which made it more difficult to grow crops on high land. Other innovations might have offset this, such as the spread of maize, mulberry plantations and the impact of work intensification as people began to work more hours a day both to increase the per hectare product and to exploit new possibilities of income such as protoindustrial activities (especially in silk manufacture). However, these did not offset the negative effects of the population increase until the 1820s, and even then, from 1835 bad harvests, silkworm and vineyard diseases caused agricultural production to drop again and it was only in the second half of the nineteenth century that technological advance was proceeding rapidly enough to support increasing incomes.

Finally, the fact that we find no evidence of preventive checks indicates that nuptiality was also being determined by noneconomic factors, i.e. societal norms. This is likely in the light of our knowledge of Italian nuptiality patterns. As Derosas (2004) notes: "the rules of family formation included other solutions than the establishment of a household on its own by the conjugal couple", so that the income restraint on marriage was not as strong as for example in England, where marriage involved the establishment of an entire new household. Malthus contrasted his ideal pattern of "individualistic marriage", which he linked to western prosperity, because it allowed for the operation of the preventive check, preventing population growth, encouraging savings and keeping the price of labor high, with "collectivistic marriage", prevalent in eastern countries. However, it has long been known

that the mechanisms through which the economy regulated nuptiality (Hajnal 1965 and 1982) were much less important in Southern European countries (Derosas 2004).

7. Conclusion

In contrast to England, preindustrial northern Italy experienced a decline in real wages over the period considered due to diminishing returns and a deterioration of the climate and agriculture. In pre-industrial Italy the preventive check was not present while the positive check was only weak. The former is in contrast to the English and Scandinavian cases but seems reasonable given the fact that Italians probably did not suffer the same income constraint on marriage as in western Europe.

In contrast to England and Scandinavia we find evidence of diminishing returns by specifying an alternative hypothesis allowing for wages to be endogenous. Extending the sample to include observations until 1881 does not change our findings and the recursive estimates of our alternative specification shows that around 1815 it becomes insignificant. Future work might try to better understand these results by extending the theoretical model of Møller and Sharp (2008) to include the additional dynamics present in Italy.

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Appendix 1: Pre-Estimation Results

A1.1 Test for stationarity

Table 4 – Augmented Dickey Fuller Test

No. of observations: 144 Null Hypothesis is: *No unit root*				
			5 lags	
Variable	**Trend**	**Test Statistic**	**Critical Value**	***p*-value**
lnw	yes	-2,563	3,444	0,2974
lnw	no	-1,597	-2,887	0,485
cbr	no	-2,032	-2,887	0,2728
cdr	no	-4,377	-2,887	0,0003
cmr	no	-5,606	-2,887	0,0918
			4 lags	
lnw	yes	-3,06	3,444	0,1162
lnw	no	-1,982	-2,887	0,2946
cbr	no	-2,073	-2,887	0,2554
cdr	no	-4,573	-2,887	0,0001
cmr	no	-3,344	-2,887	0,013
			3 lags	
lnw	yes	-3,743	3,444	0,0202
lnw	no	-2,538	-2,887	0,1065
cbr	no	-2,212	-2,887	0,2017
cdr	no	-4,288	-2,887	0,0006
cmr	no	-3,972	-2,887	0,0016

A1.2 Lag order tests

```
Tests on the significance of each variable
Variable     F-test        Value [  Prob]
lnw          F(12,344)=   14.805 [0.0000]**
cbr          F(12,344)=   11.676 [0.0000]**
cdr          F(12,344)=   3.3688 [0.0001]**
cmr          F(12,344)=   5.7209 [0.0000]**
Constant     F(4,130) =   5.2005 [0.0006]**
Trend        F(4,130) =   4.8142 [0.0012]**

Tests on the significance of each lag
Lag 3        F(16,397)=   2.0446 [0.0100]*
Lag 2        F(16,397)=   2.3518 [0.0024]**
Lag 1        F(16,397)=   13.070 [0.0000]**

Tests on the significance of all lags up to 3
Variable     F-test        Value [  Prob]           AIC           SC
Full model                                      8.40894      9.54816
Lag 3 - 3    F(16,397)=   2.0446 [0.0100]*      8.43270      9.24642
Lag 2 - 3    F(32,481)=   3.2745 [0.0000]**     8.70036      9.18859
Lag 1 - 3    F(48,502)=   14.746 [0.0000]**     11.1405      11.3033
```

A1.3 Residual analysis of model without dummies

```
Vector Portmanteau statistic for 12 lags and 147 observations: 143.514

Testing for Vector error autocorrelation from lags 1 to 4
 Chi^2(64)=   68.468 [0.3283]   and F-form F(64,448)=  0.95085 [0.5860]

Vector Normality test for Residuals
Skewness
     -0.61815     -0.19414      0.85525      0.51884
Excess kurtosis
       3.7724       2.7296       5.9826       3.7451
Skewness (transformed)
      -2.9813     -0.99690       3.9219       2.5498
Excess kurtosis (transformed)
    -0.093184     -0.60362       2.5792      0.56326
Vector Normality test:   Chi^2(8) =   39.107 [0.0000]**

Testing for Vector heteroscedasticity using squares
 Chi^2(240)=   253.54 [0.2621]   and F-form F(240,941)=  0.94056 [0.7174]

lnw         : Portmanteau(12): 10.6568
cbr         : Portmanteau(12): 3.82491
cdr         : Portmanteau(12): 7.43533
cmr         : Portmanteau(12): 17.2011
lnw         : AR 1-4 test:      F(4,129) =  0.76723 [0.5484]
cbr         : AR 1-4 test:      F(4,129) =  0.58485 [0.6742]
cdr         : AR 1-4 test:      F(4,129) =  0.80971 [0.5211]
cmr         : AR 1-4 test:      F(4,129) =   1.2231 [0.3042]
lnw         : Normality test:   Chi^2(2) =   9.3052 [0.0095]**
```

```
cbr         : Normality test:   Chi^2(2) =   4.2395 [0.1201]
cdr         : Normality test:   Chi^2(2) =   17.390 [0.0002]**
cmr         : Normality test:   Chi^2(2) =   5.8519 [0.0536]
lnw         : ARCH 1-3 test:    F(3,127) =  0.34961 [0.7895]
cbr         : ARCH 1-3 test:    F(3,127) =   1.2417 [0.2974]
cdr         : ARCH 1-3 test:    F(3,127) =  0.20908 [0.8900]
cmr         : ARCH 1-3 test:    F(3,127) =   3.7374 [0.0129]*
lnw         : hetero test:      F(24,108)=  0.45372 [0.9858]
cbr         : hetero test:      F(24,108)=   1.3714 [0.1389]
cdr         : hetero test:      F(24,108)=  0.92066 [0.5744]
cmr         : hetero test:      F(24,108)=  0.95900 [0.5247]
```

A1.4 Residual analysis of model with dummies

```
Vector Portmanteau statistic for 12 lags and 147 observations: 143.374
Testing for Vector error autocorrelation from lags 1 to 4
 Chi^2(64)=   89.742 [0.0186]*  and F-form F(64,436)=   1.2811 [0.0816]
Vector Normality test for Residuals
Skewness
     -0.42290     -0.21874      0.16065      0.50249
Excess kurtosis
       3.2265       2.7585       3.1768       3.8481
Skewness (transformed)
      -2.1122      -1.1209      0.82697       2.4766
Excess kurtosis (transformed)
     -0.18017     -0.59130      0.74568      0.91746
Vector Normality test:   Chi^2(8) =   14.315 [0.0739]

Testing for Vector heteroscedasticity using squares
 Chi^2(260)=   280.88 [0.1785]   and F-form F(260,905)=  0.92528 [0.7752]

lnw         : Portmanteau(12): 8.52297
cbr         : Portmanteau(12): 1.72734
cdr         : Portmanteau(12): 10.7463
cmr         : Portmanteau(12): 18.2454
lnw         : AR 1-4 test:      F(4,126) =  0.54084 [0.7060]
cbr         : AR 1-4 test:      F(4,126) =  0.10118 [0.9819]
cdr         : AR 1-4 test:      F(4,126) =   2.1474 [0.0788]
cmr         : AR 1-4 test:      F(4,126) =   1.0541 [0.3822]
lnw         : Normality test:   Chi^2(2) =   3.8827 [0.1435]
cbr         : Normality test:   Chi^2(2) =   3.8532 [0.1456]
cdr         : Normality test:   Chi^2(2) =   1.2744 [0.5288]
cmr         : Normality test:   Chi^2(2) =   6.6130 [0.0366]*
lnw         : ARCH 1-2 test:    F(2,126) =   1.8701 [0.1584]
cbr         : ARCH 1-2 test:    F(2,126) =  0.84317 [0.4328]
cdr         : ARCH 1-2 test:    F(2,126) =  0.40519 [0.6677]
cmr         : ARCH 1-2 test:    F(2,126) =  0.53792 [0.5853]
lnw         : hetero test:      F(26,103)=  0.74255 [0.8062]
cbr         : hetero test:      F(26,103)=   1.3544 [0.1438]
cdr         : hetero test:      F(26,103)=  0.63814 [0.9055]
cmr         : hetero test:      F(26,103)=   1.0679 [0.3920]
```

A1.5 Cointegration rank

```
Johansen tests for cointegration
Trend: rtrend                                           Number of obs =     147
Sample:  1653 - 1799                                          Lags =       3
------------------------------------------------------------------------------
                                                             5%
maximum                                       trace     critical
  rank    parms       LL       eigenvalue  statistic     value
    0      48     -591.35386           .     126.3680    62.99
    1      56     -559.89134     0.34823      63.4430    42.44
    2      62     -541.54585     0.22089      26.7520    25.32
    3      66     -531.10979     0.13237       5.8798*   12.25
    4      68     -528.16986     0.03921
------------------------------------------------------------------------------
                                                             5%
maximum                                        max      critical
  rank    parms       LL       eigenvalue  statistic     value
    0      48     -591.35386           .      62.9250    31.46
    1      56     -559.89134     0.34823      36.6910    25.54
    2      62     -541.54585     0.22089      20.8721    18.96
    3      66     -531.10979     0.13237       5.8798    12.52
    4      68     -528.16986     0.03921
------------------------------------------------------------------------------
maximum
  rank    parms       LL       eigenvalue     SBIC        HQIC        AIC
    0      48     -591.35386                  9.67516    9.095441    8.698692
    1      56     -559.89134     0.34823      9.518686   8.842348    8.379474
    2      62     -541.54585     0.22089      9.472779   8.723976    8.211508
    3      66     -531.10979     0.13237      9.466586*  8.669473*   8.123943
    4      68     -528.16986     0.03921      9.494484   8.673216    8.111155
------------------------------------------------------------------------------

Eigenvalues of companion matrix:
       real       imag    modulus
     0.9417     0.0000     0.9417
     0.7684     0.1417     0.7814
     0.7684    -0.1417     0.7814
    -0.4242    -0.4259     0.6012
    -0.4242     0.4259     0.6012
   -0.07265     0.5474     0.5522
   -0.07265    -0.5474     0.5522
     0.3815     0.3912     0.5465
     0.3815    -0.3912     0.5465
    -0.3615     0.0000     0.3615
   0.009705     0.2749     0.2751
   0.009705    -0.2749     0.2751
```

Appendix 2: Robustness Checks

A2.1 Rolling Windows Analysis

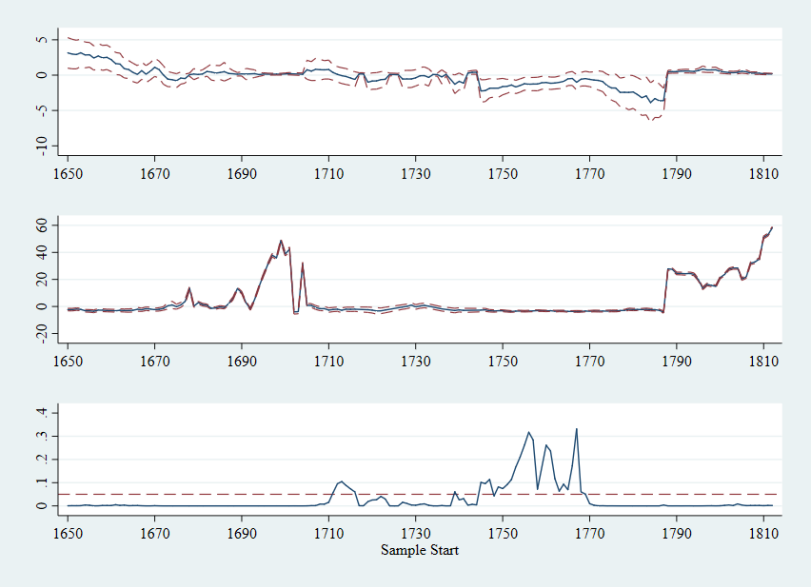

***Figure 9* – Rolling windows estimates for the specification considering homeostasis for the sample 1650-1881.**
Notes: (top panel) the point estimate of the preventive check coefficient, stated as $\boldsymbol{a_5}$, together with the 2x Standard Error limits; (middle panel) point estimate of the birth relation coefficient together with the 2x Standard Error limits; (bottom panel) the $\boldsymbol{p}$-values of the test statistic corresponding to the homeostasis hypothesis, where values below the dashed line indicate a rejection of the hypothesis at the 5 percent significance level. Size of subsamples is fixed at 70.

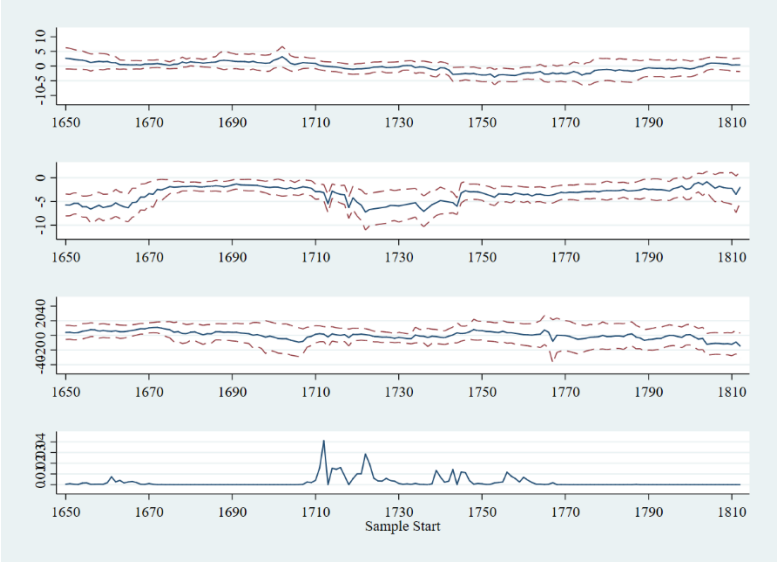

***Figure 10 – Rolling windows* estimates for the specification considering the post-Malthusian hypothesis for the sample 1650-1881.**
Notes: (top panel) the point estimate of the preventive check coefficient, stated as $\boldsymbol{a_5}$, together with the 2x Standard Error limits; (top middle panel) point estimate of the birth relation coefficient together with the 2x Standard Error limits; (middle bottom panel) the point estimate of the positive check coefficient, stated as $\boldsymbol{a_1}$, together with the 2x Standard Error limits; (bottom panel) the $\boldsymbol{p}$-values of the test statistic corresponding to the homeostasis hypothesis, where values below the dashed line indicate a rejection of the hypothesis at the 5 percent significance level. Size of subsamples is fixed at 70.

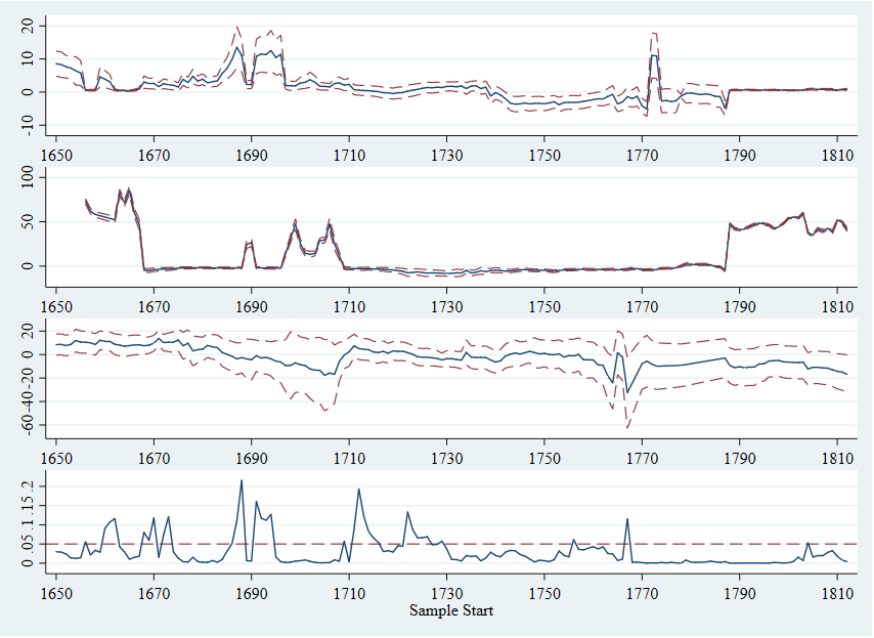

***Figure 11 – Rolling windows* estimates for the alternative specification without weakly exogenous wages for the period 1650-1799.**
Notes: (top panel) the point estimate of the preventive check coefficient, stated as $\boldsymbol{\alpha_5}$, together with the 2x Standard Error limits; (top middle panel) point estimate of the birth relation coefficient together with the 2x Standard Error limits; (middle bottom panel) the point estimate of the positive check coefficient, stated as $\boldsymbol{\alpha_1}$, together with the 2x Standard Error limits; (bottom panel) the $\boldsymbol{p}$-values of the test statistic corresponding to the homeostasis hypothesis, where values below the dashed line indicate a rejection of the hypothesis at the 5 percent significance level. Size of subsamples is fixed at 70.

A2.2 Regression results using alternative measures for income

Regression results using the inverse of wheat prices

CVAR regression results for the specification considering homeostasis for the sample 1650-1799

```
Cointegrated VAR (3) in:
[0] = lnwheat
[1] = cbr
[2] = cdr
[3] = cmr
Unrestricted variables:
[0] = Constant
[1] = D1693
[2] = D1709
[3] = D1676
Restricted variables:
[0] = Trend
Number of lags used in the analysis: 3

General cointegration restrictions:
&13=0; &14=0; &15=1;
&17=0; &18=1; &19=0; &21=0;
&22=0; &23=1; &24=-1; &25=0; &26=0;
```

```
&0=0; &1=0;
&10=0;

beta
lnwheat            0.17881      0.00000      0.00000
cbr                0.00000       1.0000       1.0000
cdr                0.00000      0.00000      -1.0000
cmr                 1.0000       9.2609      0.00000
Trend             0.0019714      0.00000      0.00000

Standard errors of beta
lnwheat            0.12526      0.00000      0.00000
cbr                0.00000      0.00000      0.00000
cdr                0.00000      0.00000      0.00000
cmr                0.00000      0.48822      0.00000
Trend            0.00075057      0.00000      0.00000

alpha
lnwheat            0.00000      0.00000  7.5206e-005
cbr                 3.6144     -0.26024     0.032385
cdr                 3.8548     -0.44148      0.53483
cmr                -0.20072      0.00000    -0.020710

Standard errors of alpha
lnwheat            0.00000      0.00000    0.0032616
cbr                0.63632     0.052382     0.038376
cdr                 1.3820      0.11440     0.080553
cmr               0.093571      0.00000     0.015108

log-likelihood    -630.369062  -T/2log|Omega|     203.966795
no. of observations       147  no. of parameters          60
rank of long-run matrix     3  no. long-run restrictions   6
beta is identified

LR test of restrictions: Chi^2(6) =   33.022 [0.0000]**
```

CVAR regression results for the specification considering the post-Malthusian hypothesis for the sample 1650-1799

```
Cointegrated VAR (3) in:
[0] = lnwheat
[1] = cbr
[2] = cdr
[3] = cmr
Unrestricted variables:
[0] = Constant
[1] = D1693
[2] = D1709
[3] = D1676
Restricted variables:
[0] = Trend
Number of lags used in the analysis: 3

General cointegration restrictions:
```

```
&13=0; &14=0; &15=1;
&17=0; &18=1; &19=0; &21=0;
&23=0; &24=1; &25=0; &26=0;


&1=0;
&3=0;
&6=0; &7=0;
&10=0; &11=0;

beta
lnwheat             2.7732      0.00000       1.6302
cbr                0.00000       1.0000      0.00000
cdr                0.00000      0.00000       1.0000
cmr                 1.0000      -4.6693      0.00000
Trend              0.017481      0.00000      0.00000

Standard errors of beta
lnwheat             0.48245      0.00000       1.5780
cbr                 0.00000      0.00000      0.00000
cdr                 0.00000      0.00000      0.00000
cmr                 0.00000      0.59718      0.00000
Trend             0.0025601      0.00000      0.00000

alpha
lnwheat            -0.10890     -0.00000    -0.014288
cbr                -0.00000     -0.23456    -0.058454
cdr                -0.00000     -0.00000     -0.57672
cmr                -0.31531     -0.00000     -0.00000

Standard errors of alpha
lnwheat            0.019819      0.00000    0.0051796
cbr                 0.00000     0.046394     0.039442
cdr                 0.00000      0.00000     0.086810
cmr                0.070954      0.00000      0.00000

log-likelihood    -618.934552  -T/2log|Omega|     215.401306
no. of observations       147  no. of parameters          58
rank of long-run matrix     3  no. long-run restrictions   8
beta is identified

LR test of restrictions: Chi^2(8) =   10.153 [0.2544]
```

Regression results using rural wages

CVAR regression results for the specification considering homeostasis for the sample 1650-1799

```
Cointegrated VAR (3) in:
[0] = lnagr
[1] = cbr
[2] = cdr
[3] = cmr
Unrestricted variables:
[0] = Constant
[1] = D1693
[2] = D1709
```

```
[3] = D1676
Restricted variables:
[0] = Trend
Number of lags used in the analysis: 3

General cointegration restrictions:
&13=0; &14=0; &15=1;
&17=0; &18=1; &19=0; &21=0;
&22=0; &23=1; &24=-1; &25=0; &26=0;

&0=0; &1=0;
&10=0;

beta
lnagr       0.85437      0.00000      0.00000
cbr         0.00000       1.0000       1.0000
cdr         0.00000      0.00000      -1.0000
cmr          1.0000       1.8812      0.00000
Trend     0.0040074      0.00000      0.00000

Standard errors of beta
lnagr       0.43457      0.00000      0.00000
cbr         0.00000      0.00000      0.00000
cdr         0.00000      0.00000      0.00000
cmr         0.00000      0.41027      0.00000
Trend     0.0016298      0.00000      0.00000

alpha
lnagr       0.00000      0.00000  -0.00057405
cbr          1.8347     -0.30222     0.036769
cdr         0.56407     -0.46349      0.53065
cmr        -0.22248      0.00000    -0.020350

Standard errors of alpha
lnagr       0.00000      0.00000    0.0016008
cbr         0.33810     0.062598     0.044027
cdr         0.69871      0.13469     0.088998
cmr         0.10333      0.00000     0.016197

log-likelihood    -547.273585  -T/2log|Omega|     287.062273
no. of observations       147  no. of parameters          60
rank of long-run matrix     3  no. long-run restrictions   6
beta is identified

LR test of restrictions: Chi^2(6) =   30.891 [0.0000]**
```

CVAR regression results for the specification considering the post-Malthusian hypothesis for the sample 1650-1799

```
Cointegrated VAR (3) in:
[0] = lnagr
[1] = cbr
[2] = cdr
[3] = cmr
Unrestricted variables:
[0] = Constant
```

```
[1] = D1693
[2] = D1709
[3] = D1676
Restricted variables:
[0] = Trend
Number of lags used in the analysis: 3

General cointegration restrictions:
&13=0; &14=0; &15=1;
&17=0; &18=1; &19=0; &21=0;
&23=0; &24=1; &25=0; &26=0;


&1=0;
&3=0;
&6=0; &7=0;
&10=0; &11=0;

beta
lnagr        5.8166      0.00000       2.8441
cbr         0.00000       1.0000      0.00000
cdr         0.00000      0.00000       1.0000
cmr          1.0000      -4.4545      0.00000
Trend      0.018872      0.00000      0.00000

Standard errors of beta
lnagr       0.97804      0.00000       2.5374
cbr         0.00000      0.00000      0.00000
cdr         0.00000      0.00000      0.00000
cmr         0.00000      0.57675      0.00000
Trend     0.0033235      0.00000      0.00000

alpha
lnagr     -0.043865     -0.00000   -0.0063090
cbr        -0.00000     -0.25720    -0.069175
cdr        -0.00000     -0.00000     -0.54262
cmr        -0.28574     -0.00000     -0.00000

Standard errors of alpha
lnagr     0.0083878      0.00000    0.0024778
cbr         0.00000     0.049855     0.041789
cdr         0.00000      0.00000     0.090287
cmr        0.061414      0.00000      0.00000

log-likelihood    -535.794541  -T/2log|Omega|     298.541316
no. of observations       147  no. of parameters          58
rank of long-run matrix     3  no. long-run restrictions   8
beta is identified

LR test of restrictions: Chi^2(8) =   7.9325 [0.4401]
```

Regression results using urban wages

CVAR regression results for the specification considering homeostasis for the sample 1650-1799

```
Cointegrated VAR (3) in:
[0] = lni
[1] = cbr
[2] = cdr
[3] = cmr
Unrestricted variables:
[0] = Constant
[1] = D1693
[2] = D1709
[3] = D1676
Restricted variables:
[0] = Trend
Number of lags used in the analysis: 3

General cointegration restrictions:
&13=0; &14=0; &15=1;
&17=0; &18=1; &19=0; &21=0;
&22=0; &23=1; &24=-1; &25=0; &26=0;

&0=0; &1=0;
&10=0;

beta
lni       -0.087885      0.00000      0.00000
cbr         0.00000       1.0000       1.0000
cdr         0.00000      0.00000      -1.0000
cmr          1.0000      -21.171      0.00000
Trend    -0.0011880      0.00000      0.00000

Standard errors of beta
lni        0.094460      0.00000      0.00000
cbr         0.00000      0.00000      0.00000
cdr         0.00000      0.00000      0.00000
cmr         0.00000      0.41486      0.00000
Trend    0.00059845      0.00000      0.00000

alpha
lni         0.00000      0.00000   -0.0019945
cbr         -5.4398     -0.31692     0.046430
cdr         -8.8474     -0.39794      0.52838
cmr        -0.14995      0.00000    -0.024802

Standard errors of alpha
lni         0.00000      0.00000    0.0016167
cbr          1.1555     0.060449     0.042163
cdr          2.4675      0.12937     0.084885
cmr        0.091676      0.00000     0.016184

log-likelihood    -548.480841  -T/2log|Omega|     285.855017
no. of observations       147  no. of parameters          60
rank of long-run matrix     3  no. long-run restrictions   6
beta is identified
```

```
LR test of restrictions: Chi^2(6) =   13.716 [0.0330]*
```

CVAR regression results for the specification considering the post-Malthusian hypothesis for the sample 1650-1799

```
Cointegrated VAR (3) in:
[0] = lni
[1] = cbr
[2] = cdr
[3] = cmr
Unrestricted variables:
[0] = Constant
[1] = D1693
[2] = D1709
[3] = D1676
Restricted variables:
[0] = Trend
Number of lags used in the analysis: 3

General cointegration restrictions:
&13=0; &14=0; &15=1;
&17=0; &18=1; &19=0; &21=0;
&23=0; &24=1; &25=0; &26=0;


&1=0;
&3=0;
&6=0; &7=0;
&10=0; &11=0;

beta
lni           2.8067      0.00000       1.8235
cbr          0.00000       1.0000      0.00000
cdr          0.00000      0.00000       1.0000
cmr           1.0000      -4.3431      0.00000
Trend       0.019772      0.00000      0.00000

Standard errors of beta
lni          0.92926      0.00000       1.5306
cbr          0.00000      0.00000      0.00000
cdr          0.00000      0.00000      0.00000
cmr          0.00000      0.60811      0.00000
Trend      0.0055634      0.00000      0.00000

alpha
lni        -0.032200     -0.00000   -0.0029804
cbr         -0.00000     -0.24665    -0.077574
cdr         -0.00000     -0.00000     -0.53828
cmr         -0.27423     -0.00000     -0.00000

Standard errors of alpha
lni        0.0090779      0.00000    0.0025158
cbr          0.00000     0.050700     0.041118
```

```
cdr          0.00000      0.00000     0.088479
cmr         0.065033      0.00000      0.00000

log-likelihood    -545.668738  -T/2log|Omega|      288.66712
no. of observations       147  no. of parameters          58
rank of long-run matrix     3  no. long-run restrictions   8
beta is identified

LR test of restrictions: Chi^2(8) =   8.0914 [0.4246]
```

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